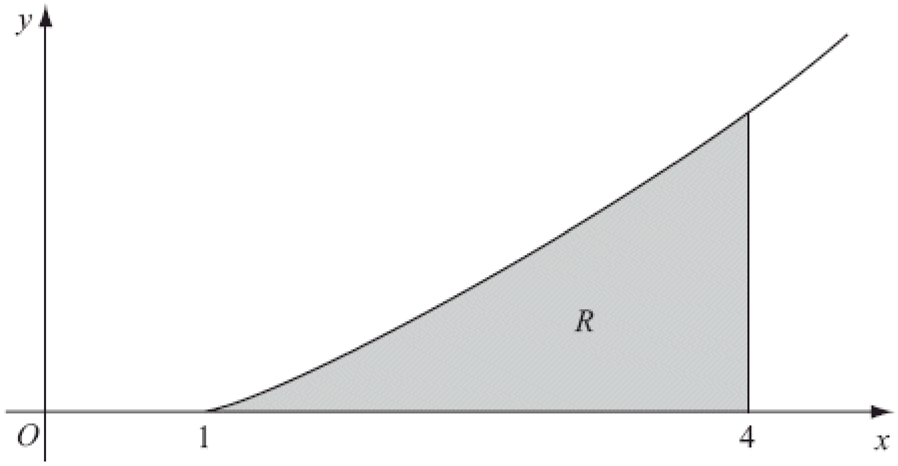
1.



The diagram above shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

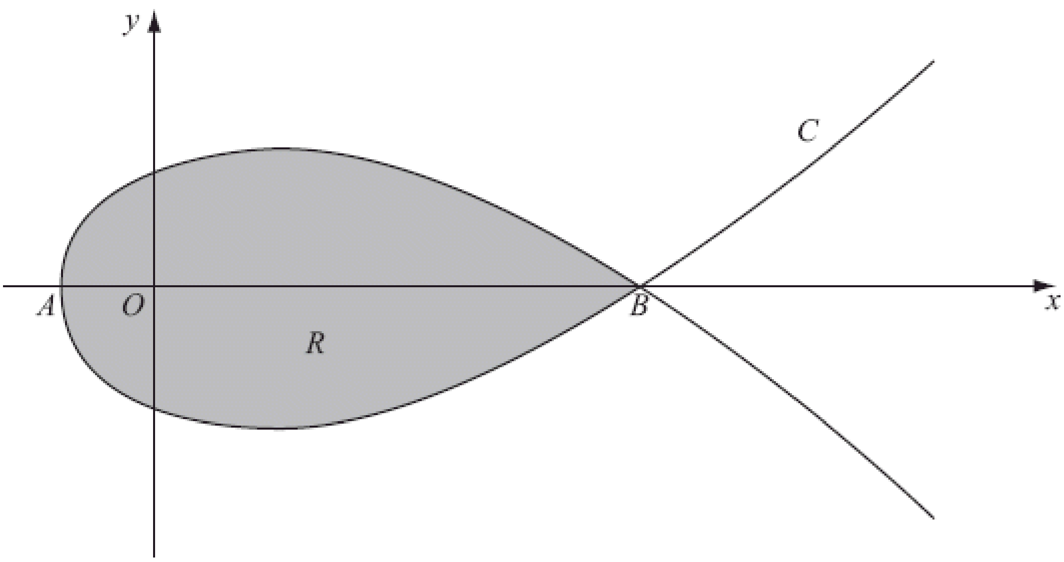
(c) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

(Total 13 marks)

2.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

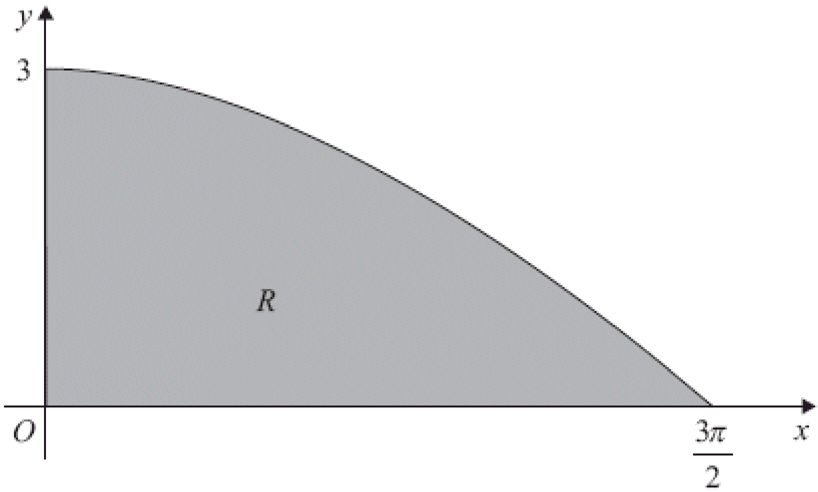
(3)

The region R, as shown shaded in the diagram above, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)
Total 9 marks)

3.



The diagram above shows the finite region *R* bounded by the *x*-axis, the *y*-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right), 0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of *x* and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of *y* to 5 decimal places.

(1)

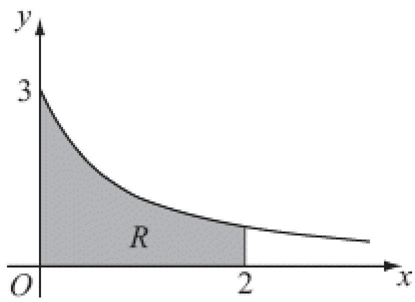
(b) Using the trapezium rule, with all the values of *y* from the completed table, find an approximation for the area of *R*, giving your answer to 3 decimal places.

(4)

(c) Use integration to find the exact area of *R*.

(3)
(Total 8 marks)

4.



The diagram above shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in the diagram above.

(a) Use integration to find the area of R.

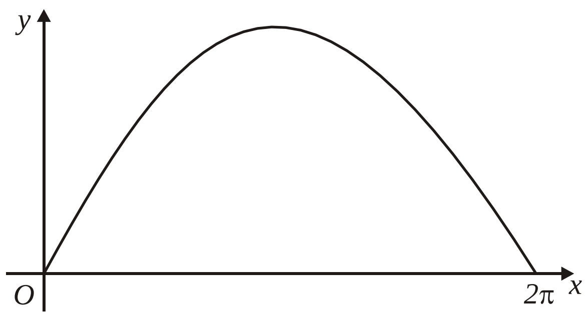
(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)
(Total 9 marks)

5.



The curve with equation, $y = 3\sin\frac{x}{2}, 0 \le x \le 2\pi$, is shown in the figure above. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

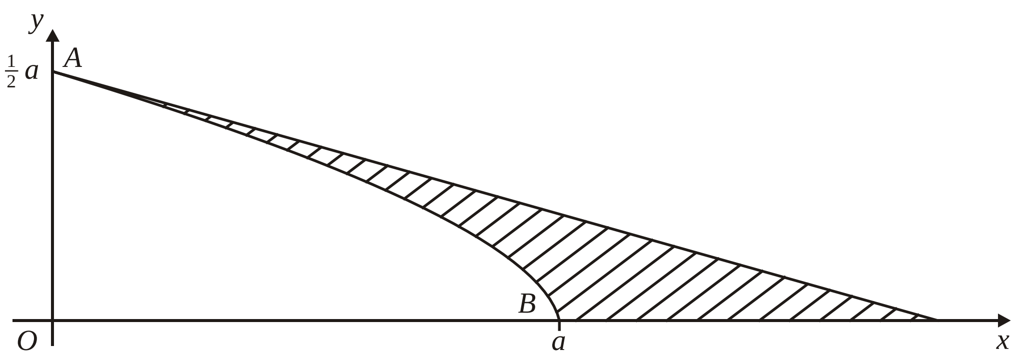
(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)
(Total 9 marks)

6.



The curve shown in the figure above has parametric equations

$$x = a\cos 3t,\ \ y = a\sin t, \quad 0 \le t \le \frac{\pi}{6}.$$

The curve meets the axes at points A and B as shown.

The straight line shown is part of the tangent to the curve at the point A.

Find, in terms of a,

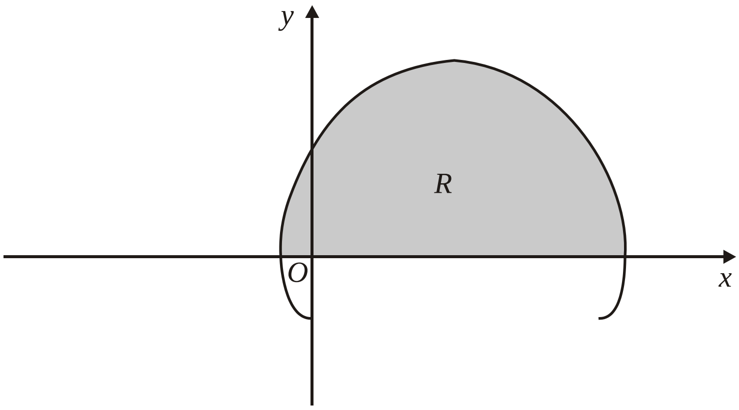
(a) an equation of the tangent at A,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at A and the x-axis, shown shaded in the figure above.

(9)
(Total 15 marks)

7.



The curve shown in the figure above has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \le t \le 2\pi$$

(a) Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in the figure above.

(b) Show that the area of R is given by the integral

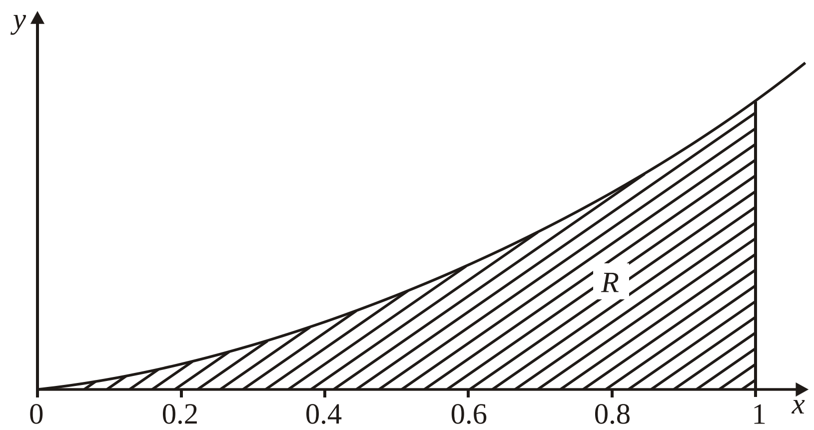
$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)
(Total 12 marks)

8.



The diagram shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in the diagram.

(a) Use integration to find the exact value of the area for R.

(5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

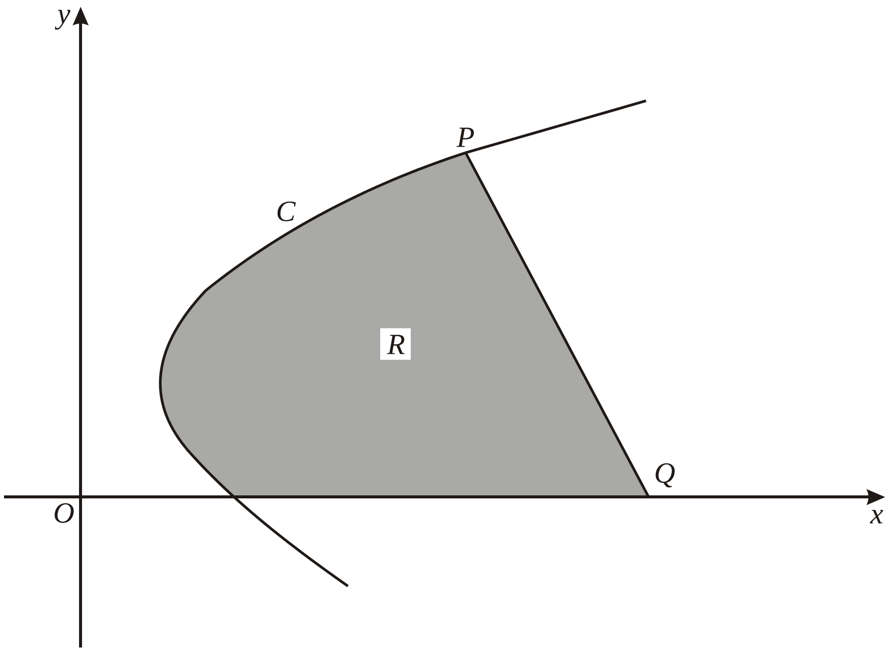
x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

9.



The diagram shows a sketch of part of the curve C with parametric equations

$$x = t^2 + 1, \qquad y = 3(1 + t).$$

The normal to C at the point $P(5, 9)$ cuts the x-axis at the point Q, as shown in the diagram.

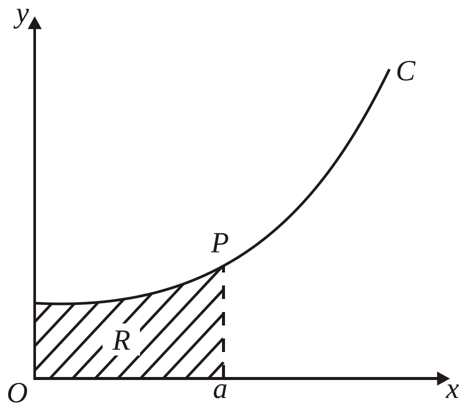
(a) Find the x-coordinate of Q.

(6)

(b) Find the area of the finite region R bounded by C, the line PQ and the x-axis.

(9)
(Total 15 marks)

10.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 3t \sin t,\ y = 2 \sec t, \qquad 0 \le t < \frac{\pi}{2}.$$

The point $P(a, 4)$ lies on C.

(a) Find the exact value of a.

(3)

The region R is enclosed by C, the axes and the line $x = a$ as shown in the diagram above.

(b) Show that the area of R is given by

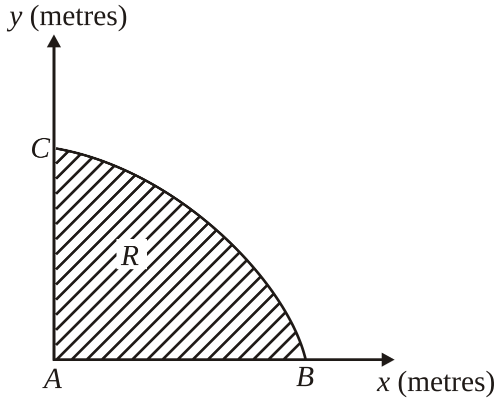
$$6\int_0^{\frac{\pi}{3}} (\tan t + t)\,dt.$$

(4)

(c) Find the exact value of the area of R.

(4)
(Total 11 marks)

11.



The diagram above shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates (0, 0), $(3\pi^2, 0)$ and (0, 30) respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(a) Find an estimate for the area of the cross-section R using this approximation.

(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30 \sin 2t, \quad \frac{\pi}{4} \le t \le \frac{\pi}{2}.$$

(b) Find the exact area of the cross-section R.

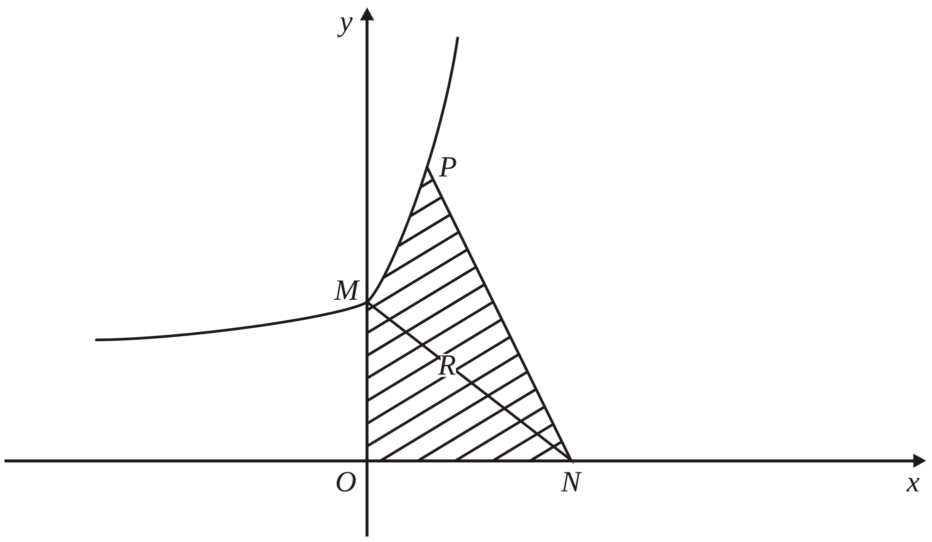
(7)

(c) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (a).

(2)

(Total 10 marks)

12.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)
(Total 12 marks)

13.

$$g(x) = \frac{5x + 8}{(1 + 4x)(2 - x)}.$$

(a) Express g(x) in the form $\frac{A}{(1+4x)} + \frac{B}{(2-x)}$, where A and B are constants to be found.

(3)

The finite region R is bounded by the curve with equation y = g(x), the coordinate axes and the line $x = \frac{1}{2}$.

(b) Find the area of R, giving your answer in the form $a \ln 2 + b \ln 3$.

(7)
(Total 10 marks)

1.	(a)	1.386, 2.291	awrt 1.386, 2.291	B1 B1	2
	(b)	$A \approx \frac{1}{2} \times 0.5(\ldots)$		B1	
		= ... (0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)		M1	
		= 0.25(0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)	ft their (a)	A1ft	
		=0.25×29.477 ... ≈7.37	cao	A1	4
	(c)	(i) $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$		M1 A1	
		$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$			
		$= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+C)$		M1 A1	
		(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1^4 = (8 \ln 4 - 4) - \left(-\frac{1}{4} \right)$		M1	
		$=8\ln 4 - \frac{15}{4}$			
		$=8(2\ln2) - \frac{15}{4}$	ln 4 = 2ln 2 seen or implied	M1	
		$= \frac{1}{4}(64 \ln 2 - 15)$	a = 64, b = –15	A1	7

[13]

2.	(a)	$y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$			
		t = 0, 3, –3	Any one correct value	B1	
		At t = 0, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
		At t = 3, $x = 5(3)^2 - 4 = 41$			
		(At $t = -3$, $x = 5(-3)^2 - 4 = 41$)			
		At A, x = –4; at B, x = 41	Both	A1	3

	(b)	$\frac{dx}{dt} = 10t$	Seen or implied	B1	
		$\int y\,dx = \int y\frac{dx}{dt}dt = \int t(9-t^2)10t\,dt$		M1 A1	
		$= \int (90t^2 - 10t^2)dt$			
		$\left[\frac{90t^3}{3} - \frac{10t^5}{5}\right]_0^3 = 30\times 3^3 - 2\times 3^5 (= 324)$		M1	
		$A = 2\int y\,dx = 648$ (units^2)		A1	6

[9]

3.	(a)	1.14805	awrt 1.14805	B1	1
	(b)	$A \approx \frac{1}{2}\times\frac{3\pi}{8}(...)$		B1	
		= ... (3 + 2(2.77164 + 2.12132 + 1.14805) + 0)	0 can be implied	M1	
		$= \frac{3\pi}{16}$ (3 + 2(2.77164 + 2.12132 + 1.14805))	ft their (a)	A1ft	
		$\frac{3\pi}{16}$ × 15.08202 ... = 8.884	cao	A1	4
	(c)	$\int 3\cos\left(\frac{x}{3}\right)dx = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$		M1 A1	
		$= 9\sin\left(\frac{x}{3}\right)$			
		$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$	cao	A1	3

[8]

4.	(a)	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}dx$			
		Integrating $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.		M1	

$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$ Correct integration. Ignore limits. A1

$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$

$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round. M1

$= \frac{9}{2} - \frac{3}{2} = \underline{3}(\text{units})^2$ $\underline{3}$ $\underline{\text{A1}}$ 4

(Answer of 3 with no working scores M0A0M0A0.)

(b) Volume = $\pi\int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 \mathrm{d}x$ Use of $V = \pi\int y^2\,\mathrm{d}x$ .. Can be implied. Ignore limits and dx. B1

$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$

$= (\pi)\left[\frac{9}{4}\ln|1+4x|\right]_0^2$ $\pm k\ln|1+4x|$ M1

$\frac{9}{4}\ln|1+4x|$ A1

$= (\pi)\left[(\frac{9}{4}\ln 9) - (\frac{9}{4}\ln 1)\right]$ Substitutes limits of 2 and 0 and subtracts the correct way round. dM1

Note that ln1 can be implied as equal to 0.

So Volume = $\underline{\frac{9}{4}\pi\ln 9}$ $\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$ A1 oe isw 5

Note the answer must be a one term exact value. Note that $\frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

Note, also you can ignore subsequent working here.

[9]

5. (a) Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\mathrm{d}x$

$= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{-2\pi}$

Integrating 3sin($\frac{x}{2}$) to give k cos ($\frac{x}{2}$) with k ≠ 1. Ignore limits. M1

$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$

$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ A1 oe

$= \left[-6(-1)\right]-\left[-6(1)\right]=6+6=\underline{12}$ A1 cao 3

(Answer of 12 with no working scores M0A0A0.)

(b) Volume = $\underline{\pi\int_0^{2\pi}\left(3\sin\left(\frac{x}{2}\right)\right)^2 \quad dx}=9\pi\int_0^{2\pi}\sin^2\left(\frac{x}{2}\right)dx$

Use of $V=\pi\int y^2\,dx$.

Can be implied. Ignore limits. M1

[NB: $\cos 2x = \pm 1 \pm 2\sin^2 x$ gives $\sin^2 x \;\; x = \frac{1-\cos 2x}{2}$]

[NB: $\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)$ gives $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$] M1

Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ *or the Double Angle Formula for* $\sin^2 x$

$\therefore$ Volume $= 9(\pi)\int_0^{2\pi}\left(\frac{1-\cos x}{2}\right)dx$ A1

Correct expression for Volume
Ignore limits and π.

$= \frac{9(\pi)}{2}\int_0^{2\pi}(\underline{1-\cos x})\,dx$

$= \frac{9(\pi)}{2}\left[x-\sin x\right]_0^{2\pi}$

Integrating to give $\pm ax \pm b\sin x$; depM1;

Correct integration

$k - k\cos x \to kx - k\sin x$ A1

$= \frac{9\pi}{2}\left[(2\pi-0)-(0-0)\right]$

$= \frac{9\pi}{2}(2\pi)=\underline{9\pi^2}$ or $\underline{88.8264.}$ A1 cso 3

Use of limits to give either $9\pi^2$ *or awrt 88.8*
Solution must be completely correct. No flukes allowed.

[6]

6.	(a)	$\frac{dx}{dt} = -3a\sin 3t$, $\quad \frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$	M1 A1	
		When $x = 0$, $t = \frac{\pi}{6}$	B1	
		Gradient is $-\frac{\sqrt{3}}{6}$	M1	
		Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$	M1 A1	6
	(b)	Area beneath curve is $\int a\sin t\,(-3a\sin 3t)dt$	M1	
		$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$	M1	
		$\frac{3a^2}{2}[\frac{1}{2}\sin 2t - \frac{1}{4}\sin 4t]$	M1 A1	
		Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$	A1	
		Area of triangle beneath tangent is $\frac{1}{2} \times \frac{a}{2} \times \sqrt{3}a = \frac{\sqrt{3}a^2}{4}$	M1 A1	
		Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$	A1	9
	N.B.	The integration of the product of two sines is worth 3 marks (lines 2 and 3 of to part (b)) If they use parts		
		$\int \sin t \sin 3t dt = -\cos t \sin 3t + \int 3\cos 3t \cos t dt$	M1	
		$= -\cos t \sin 3t + 3\cos 3t \sin t + \int 9\sin 3t \sin t dt$		
		$8I = \cos t \sin 3t - 3\cos 3t \sin t$	M1 A1	

[15]

7	(a)	Solves y = 0 $\Rightarrow$ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$		
		(need both for A1)	M1 A1	2
		Or substitutes **both** values of t and shows that $y = 0$		

(b) $\frac{dx}{dt} = 1 - 2\cos t$ M1 A1

Area = $\int y\,dx = \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)\ (1 - 2\cos t)dt$

$= \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)^2\ dt$ AG B1 3

(c) Area $= \int 1 - 4\cos t + 4\cos^2 t\ dt$ 3 terms M1

$\int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula) M1

$= \int 3 - 4\cos t + 2\cos 2t dt$ M1 A1

$= [3t - 4\sin t + \sin 2t]$ M1 A1

Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts. M1

$= 4\pi + 3\sqrt{3}$ A1A1 7

[12]

8. (a) $\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$ M1 A1

Attempting parts in the right direction

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$ A1

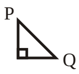
$\left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}e^2$ M1 A1 5

(b) $x = 0.4 \Rightarrow y \approx 0.89022$
$x = 0.8 \Rightarrow y \approx 3.96243$ B1 1

Both are required to 5.d.p.

	(c)	$I \approx \frac{1}{2} \times 0.2 \times [\ldots]$	B1	
		$\approx \ldots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$	M1 A1ft	
		ft their answers to (b)		
		$\approx 0.1 \times 21.67522$		
		≈ 2.168	cao A1	4
		Note: $\frac{1}{4}+\frac{1}{4}e^2 \approx 2.097\ldots$		

[10]

9.	(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$	M1	
		Gradient of normal is $-\frac{2t}{3}$	M1	
		At P $t = 2$	B1	
		$\therefore$ Gradient of normal @ P is $-\frac{4}{3}$	A1	
		Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$	M1	
		Q is where $y = 0 \;\therefore\; x = \frac{27}{4} + 5 = \underline{\frac{47}{4}}$ (o.e.)	A1	6
	(b)	Curved area = $\int y\,\mathrm{d}x = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	M1	
		$= \int 3(1 + b).2t\,\mathrm{d}t$	A1	
		$= [3t^2 + 2t^3]$	M1A1	
		Curve cuts x-axis when $t = -1$	B1	
		Curved area = $[3t^2 + 2t^3]_{-1}^{2} = (12 + 16) - (3 - 2)$ (= 27)	M1	
		Area of  triangle = $\frac{1}{2}((a) - 5) \times 9$ (= 30.375)	M1	
		Total area of R = curved area + Δ	M1	
		= 57.375 or AWRT <u>57.4</u>	A1	9

[15]

10. (a) $4 = 2\sec t \Rightarrow \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$ M1, A1

$\therefore\ a = 3\times\frac{\pi}{3}\times\sin\frac{\pi}{3} = \underline{\frac{\pi\sqrt{3}}{2}}$ B1 3

(b) $A = \int_0^a y\,dx = \int y\frac{dx}{dt}\,dt$ M1

Change of variable

$= \int 2\sec t \times [3\sin t + 3t\cos t]\,dt$ M1

Attempt $\frac{dx}{dt}$

$= \int_0^{\frac{\pi}{3}} (6\tan t, + 6t)\,dt$ (*) A1, A1cso 4

Final A1 requires limit *<u>stated</u>*

(c) $A = [6\ln\sec t + 3t^2]_0^{\frac{\pi}{3}}$ M1, A1

Some integration (M1) both correct (A1) ignore lim.

$= (6\ln 2 + 3\times\frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$ M1

$= \underline{6\ln 2 + \frac{\pi^2}{3}}$ A1 4

[11]

11. (a) Area of triangle = $\frac{1}{2}\times 30\times 3\pi^2$ (= 444.132) B1 1

Accept 440 or 450

(b) **Either** Area shaded = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30\sin 2t.32t\,dt$ M1 A1

$= [-480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ M1 A1

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ M1 A1 7

or $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 60\cos 2t.(16t^2 - \pi^2)\,\mathrm{d}t$ M1 A1

$= [(30\sin 2t\,(\pi^2 - 16t^2) - 480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ M1 A1

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ M1 A1 7

(c) Percentage error = $\dfrac{240(\pi - 1) - \text{estimate}}{240(\pi - 1)} \times 100 = 13.6\%$ M1 A1 2

(Accept answers in the range 12.4% to 14.4%)

[10]

12. (a) M is (0, 7) B1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2e^x$ M1

Attempt $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

$\therefore$ gradient of normal is $-\frac{1}{2}$ M1

ft their y′(0) or $= -\dfrac{1}{2}$

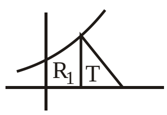
(Must be a number)

$\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $\underline{x + 2y - 14 = 0}$

$\underline{x + 2y = 14}$ o.e. A1 4

(b) $y = 0$, $x = 14$ $\therefore$ $\underline{N \text{ is } (14, 0)}$ (*) B1 cso 1

(c)



$\int (2e^x + 5)\,\mathrm{d}x = [2e^x + 5x]$ M1

some correct $\int$

$R_1 = \int_0^{\ln 4} (2e^x + 5)\,\mathrm{d}x = (2 \times 4 + 5\ln 4) - (2 + 0)$ M1 *limits used*

= 6 + 5 ln 4 A1

$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$ B1

Area of T

$T = 13(7 - \ln 2)$; $R_1 = 6 + 10 \ln 2$ B1

Use of ln 4 = 2ln 2

$R = T + R_1$, $\underline{R = 97 - 3 \ln 2}$ M1, A1 7

[12]

13. (a) $A(2 - x) + B(1 + 4x) = 5x + 8$ M1

$x = 2 \quad 9B = 18 \quad \Rightarrow B = 2$ A1

$x = -\frac{1}{4} \quad \frac{9A}{4} = \frac{27}{4} \quad \Rightarrow A = 3$ A1 3

(b) Area = $\int_0^{\frac{1}{2}} g(x)\, dx$ + (attempt to integrate)

$= 3\int \frac{dx}{(1+4x)} + 2\int \frac{dx}{(2-x)}$

$= \frac{3}{4}\left[\ln(1+4x)\right]_0^{\frac{1}{2}} - 2\left[\ln(2-x)\right]_0^{\frac{1}{2}}$ A1 A1

$= \frac{3}{4} \ln 3 - 2 \ln\left(\frac{3}{2}\right) + 2 \ln 2$ M1 A1 A1

$= \frac{3}{4} \ln 3 - 2 \ln 3 + 2 \ln 2 + 2 \ln 2$

$= 4 \ln 2 - \frac{5}{4} \ln 3$ A1 7

[10]

1. Nearly all candidates gained both marks in part (a). As is usual, the main error seen in part (b) was finding the width of the trapezium incorrectly. There were fewer errors in bracketing than had been noted in some recent examinations and nearly all candidates gave the answer to the specified accuracy. The integration by parts in part (c) was well done and the majority of candidates had been well prepared for this topic.

Some failed to simplify $\int \frac{x^2}{2} \times \frac{1}{x} \mathrm{d}x$ to $\int \frac{x}{2} \mathrm{d}x$ and either gave up or produced $\frac{\frac{1}{3}x^3}{x^2}$.

In evaluating the definite integral some either overlooked the requirement to give the answer in the form $\frac{1}{4}(a \ln 2 + b)$ or were unable to use the appropriate rule of logarithms correctly.

2. Part (a) was well done. The majority of candidates correctly found the x-coordinates of A and B, by putting $y = 0$, solving for t and then substituting in $x = 5t^2 – 4$. Full marks were common. Part (b) proved difficult. A substantial minority of candidates failed to substitute for the dx when substituting into $\int y \, \mathrm{d}x$ or used $\frac{\mathrm{d}t}{\mathrm{d}x}$ rather than $\frac{\mathrm{d}y}{\mathrm{d}x}$. A surprising feature of the solutions seen was the number of candidates who, having obtained the correct $\int t\left(9 - t^2\right)10t \, \mathrm{d}t$, were unable to remove the brackets correctly to obtain $\int \left(90t^2 - 10t^4\right)\mathrm{d}t$. Weaknesses in elementary algebra flawed many otherwise correct solutions. Another source of error was using the x-coordinates for the limits when the variable in the integral was t. At the end of the question, many failed to realise that $\int_0^3 \left(90t^2 - 10t^4\right)\mathrm{d}t$ gives only half of the required area.

Some candidates made either the whole of the question, or just part (b), more difficult by eliminating parameters and using the cartesian equation. This is a possible method but the indices involved are very complicated and there were very few successful solutions using this method.

3. Most candidates could gain the mark in part (a) although 2.99937, which arises from the incorrect angle mode, was seen occasionally. The main error seen in part (b) was finding the width of the trapezium incorrectly, $\frac{3\pi}{10}$ being commonly seen instead of $\frac{3\pi}{8}$. This resulted from confusing the number of values of the ordinate, 5, with the number of strips, 4. Nearly all candidates gave the answer to the specified accuracy. In part (c), the great majority of candidates recognised that they needed to find $\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x$ and most could integrate correctly. However sin x, 9sin x, 3sin $\left(\frac{x}{3}\right)$, $-9\sin\left(\frac{x}{3}\right)$, $-\sin\left(\frac{x}{3}\right)$ and $-3\sin\left(\frac{x}{3}\right)$ were all seen from time to time. Candidates did not seem concerned if their answers to part (b) and part (c) were quite different, possibly not connecting the parts of the question. Despite these difficulties, full marks were common and, generally, the work on these topics was sound.

4. Q2 was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), a significant minority of candidates tried to integrate $3(1+4x)^{\frac{1}{2}}$. Many candidates, however, correctly realised that they needed to integrate $3(1+4x)^{-\frac{1}{2}}$. The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $k(1+4x)^{\frac{1}{2}}$. Few candidates applied incorrect limits to their integrated expression. A noticeable number of candidates, however, incorrectly assumed a subtraction of zero when substituting for $x = 0$ and so lost the final two marks for this part. A minority of candidates attempted to integrate the expression in part (a) by using a substitution. Of these candidates, most were successful.

In part (b), the vast majority of candidates attempted to apply the formula $\pi\int y^2\,dx$, but a few of them were not successful in simplifying y^2. The majority of candidates were able to integrate $\frac{9}{1+4x}$ to give $\frac{9}{4}\ln|1+4x|$. The most common error at this stage was for candidates to omit dividing by 4. Again, more candidates were successful in this part in substituting the limits correctly to arrive at the exact answer of $\frac{9}{4}\pi\ln 9$. Few candidates gave a decimal answer with no exact term seen and lost the final mark.

5. In part (a), most candidates realised that to find the shaded area they needed to integrate $3\sin(\frac{x}{2})$ with respect to x, and the majority of them produced an expression involving $\cos(\frac{x}{2})$; so gaining the first method mark. Surprisingly a significant number of candidates were unable to obtain the correct coefficient of -6, so thereby denying themselves of the final two accuracy marks. Most candidates were able to use limits correctly, though some assumed that cos0 is zero.
In part (b), whilst most candidates knew the correct formula for the volume required, there were numerous errors in subsequent work, revealing insufficient care in the use or understanding of trigonometry. The most common wrong starting point was for candidates to write y^2as $3\sin^2\left(\frac{x^2}{4}\right)$, $9\sin^2(\frac{x^2}{4})$or $3\sin^2(\frac{x}{2})$. Although some candidates thought that they could integrate $\sin^2(\frac{x}{2})$directly to give them an incorrect expression involving $\sin^3(\frac{x}{2})$, many realised that they needed to consider the identity $\cos 2A \equiv 1 - 2\sin^2 A$ and so gained a method mark. At this stage, a significant number of candidates found difficultly with rearranging this identity and using the substitution A= $\frac{x}{2}$ to give the identity $\sin^2\left(\frac{x}{2}\right)\equiv\frac{1-\cos x}{2}$. Almost all of those candidates who were able to substitute this identity into their volume expression proceeded to correct integration and a full and correct solution.
There were, however, a significant minority of candidates who used the method of integration by parts in part (b), but these candidates were usually not very successful in their attempts.

6. This question proved a significant test for many candidates with fully correct solutions being rare. Many candidates were able to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$, although confusing differentiation with integration often led to inaccuracies. Some candidates attempted to find the equation of the tangent but many were unsuccessful because they failed to use $t=\frac{\pi}{6}$ in order to find the gradient as $-\frac{\sqrt{3}}{6}$.

Those candidates who attempted part (b) rarely progressed beyond stating an expression for the area under the curve. Some attempts were made at integration by parts, although very few candidates went further than the first line. It was obvious that most candidates were not familiar with integrating expressions of the kind $\int \sin at \sin bt \, dt$. Even those who were often spent time deriving results rather than using the relevant formula in the formulae book.

Those candidates who were successful in part (a) frequently went on to find the area of a triangle and so were able to gain at least two marks in part (b).

7. The majority of candidates gained the marks in part (a) and a good proportion managed to produce the given result in part (b). Some candidates suggested that the area of R was $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} y^2 \, dt$, which made the question rather trivial; although that happened to be true here as $y = \frac{dx}{dt}$, working was needed to produce that statement.

The integration in part (c), although well done by good candidates, proved a challenge for many; weaker candidates integrating $(1\text{-}2\cos t)^2$ as $\frac{(1-2\cos t)^3}{3}$ or something similar. It may have been that some candidates were pressed for time at this point but even those who knew that a cosine double-angle formula was needed often made a sign error, forgot to multiply their expression for $\cos^2 t$ by 4, or even forgot to integrate that expression. It has to be mentioned again that the limits were sometimes used as though $\pi = 180$,

so that $\left[3t - 4\sin t + \sin 2t\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$ became [900 – …….] – [180 – ….].

8. Those who recognised that integration by parts was needed in part (a), and these were the great majority, usually made excellent attempts at this part and, in most cases, the indefinite integral was carried out correctly. Many had difficulty with the evaluating the definite integral. There were many errors of sign and the error $e^0 = 0$ was common. The trapezium rule was well known, although the error of thinking that 6 ordinates gave rise to 6 strips, rather than 5, was often seen and some candidates lost the final mark by not giving the answer to the specified accuracy.

9. Many good attempts at part (a) were seen by those who appreciated $t = 2$ at P, or by those using the Cartesian equation of the curve. Algebraic errors due to careless writing led to a loss of accuracy throughout the question, most commonly

$\frac{3}{2t} \to \frac{3}{2}t \to \frac{3t}{2}$

Part (b) undoubtedly caused candidates extreme difficulty in deciding which section of the shaded area *R* was involved with integration. The majority set up some indefinite integration and carried this out well. Only the very able sorted out the limits satisfactorily. Most also evaluated the area of either a triangle or a trapezium and combined, in some way, this with their integrand, demonstrating to examiners their overall understanding of this situation.

10. Part (a) was often answered well but some candidates who worked in degrees gave the final answer as $90\sqrt{3}$. Part (b) proved more challenging for many; some did not know how to change the variable and others failed to realize that $\frac{\mathrm{d}x}{\mathrm{d}t}$ required the chain rule. Most candidates made some progress in part (c) although a surprising number thought that $\int \tan t \ \mathrm{d}t$ was $\sec^2 t$. The examiners were encouraged to see most candidates trying to give an exact answer (as required) rather than reaching for their calculators.

11. This was found to be the most difficult question on the paper. Some excellent candidates did not appear to have learned how to find the area using parametric coordinates and could not even write down the first integral. A few candidates used the formula $\frac{1}{2}\int x\frac{dy}{dt} - \frac{1}{2}\int y\frac{dx}{dt}$ from the formula book. The integration by parts was tackled successfully, by those who got to that stage and there were few errors seen. The percentage at the end of the question was usually answered well by the few who completed the question.

12. Whilst the majority of answers to part (a) were fully correct, some candidates found difficulties here. A small number failed to find the coordinates of *M* correctly with (0, 5) being a common mistake. Others knew the rule for perpendicular gradients but did not appreciate that the gradient of a normal must be numerical. A few students did not show clearly that the gradient of the curve at $x = 0$ was found from the derivative, they seemed to treat $y = 2e^x + 5$ and assumed the gradient was always 2. Some candidates failed to obtain the final mark in this section because they did not observe the instruction that *a, b* and *c* must be integers.

For most candidates part (b) followed directly from their normal equation. It was disappointing that those who had made errors in part (a) did not use the absence of $n = 14$ here as a pointer to check their working in the previous part. Most preferred to invent all sorts of spurious reasons to justify the statement.

Many candidates set out a correct strategy for finding the area in part (c). The integration of the curve was usually correct but some simply ignored the lower limit of 0. Those who used the simple "half base times height" formula for the area of the triangle, and resisted the lure of their calculator, were usually able to complete the question. Some tried to find the equation of *PN* and integrate this but they usually made no further progress. The demand for exact answers proved more of a challenge here than in 6(c) but many candidates saw clearly how to simplify $2e^{\ln 4}$ and convert ln 4 into 2 ln 2 on their way to presenting a fully correct solution.

13. No Report available for this question.